HW8 solution

7.4.1 The likelihood function is given by $L(\lambda \mid x_1, ..., x_n) = \lambda^n \prod (1+x_i)^{-\lambda-1}$. The prior distribution has density given by $\pi(\lambda) = \beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda} / \Gamma(\alpha)$. The posterior density is then proportional to $\lambda^{n+\alpha-1} \prod (1+x_i)^{-\lambda} e^{-\beta\lambda} =$
$\lambda^{n+\alpha-1} \exp(-\lambda \ln(\prod(1+x_i))) e^{-\beta\lambda} = \lambda^{n+\alpha-1} \exp[-\lambda(\ln(\prod(1+x_i)) + \beta)]$, and so the posterior is a Gamma$(n+\alpha, \ln(\prod(1+x_i)) + \beta)$ distribution. Hence, this is a conjugate family.

7.4.2 The likelihood function is given by $L(\theta \mid x_1, ..., x_n) = \theta^{-n} I_{[x_{(n)}, \infty)}(\theta)$. The prior distribution has density given by $\pi(\theta) = \theta^{-\alpha} I_{[\beta,\infty)}(\theta) / (\alpha-1)\beta^{\alpha-1}$, where $\alpha \geq 1$ and $\beta > 0$. The posterior density is then proportional to $\theta^{-n-\alpha} I_{[x_{(n)},\infty)}(\theta) I_{[\beta,\infty)}(\theta) = \theta^{-n-\alpha} I_{[\max\{x_{(n)},\beta\},\infty)}$, which is of the same form as the family of priors and so this is a conjugate family for this problem.

9.1.5 By grouping the data into five equal intervals each having length 0.2, the expected counts for each interval are $np_i = 4$, and the observed counts are given in the following table.

Interval	Count
$(0.0, 0.2]$	4
$(0.2, 0.4]$	7
$(0.4, 0.6]$	3
$(0.6, 0.8]$	4
$(0.8, 1]$	2

The Chi-squared statistic is equal to 3.50 and the P-value is given by $(X^2 \sim \chi^2(4))$ $P(X^2 \geq 3.5) = 0.4779$ Therefore, we have no evidence against the Uniform model being correct.

9.1.6 First note that if the die is fair, the expected number of counts for each possible outcome is 166.667. The Chi-squared statistic is equal to 9.5720 and the P-value is given by $(X^2 \sim \chi^2(5))$ $P(X^2 \geq 9.5720) = .08831$. Therefore, we have some evidence that the die might not be fair. The standardized residuals are given in the following table.

i	1	2	3	4	5	6
r_i	-0.069541	0.214944	-0.467818	-0.316093	0.309772	0.328737

None of these look unusual.